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Stimulated waves of electric and magnetic fields in a two-dimensional semi-infinite Josephson junction

A Grigorov, D Ouroushev and N Martinov

Department of Condensed Matter Physics, Faculty of Physics, Sofia University, boul. J Baucher 5, 1126 Sofia, Bulgaria

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Abstract. A two-dimensional semi-infinite Josephson junction without damping is considered. Its interaction with an external oscillating electromagnetic field in the form of a running wave with a phase velocity equal to the Swihart velocity is investigated. The results obtained are based on an exact solution of the (2+1)-dimensional sine–Gordon equation, depending on an arbitrary function. The boundary conditions on the interface are provided by an external time-varying electric field consistent with the exact solution. Under these conditions, an electromagnetic structure arises inside the junction. It is shown how the existence of this formation may be proved experimentally. A method for measuring the Swihart velocity is proposed.

1. Introduction

It is well known that when the size of a plane Josephson junction is much greater than the Josephson penetration depth λ_J, a macroscopic description becomes possible. Then the junction is considered as a solid body in which may exist a macroscopic magnetic field $\boldsymbol{H}$ and electric field $\boldsymbol{E}$ [1–4].

When the electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{H}$ are sufficiently weak or the conditions

$$eV \ll \Delta \qquad |\boldsymbol{H}| \ll H_{\mathrm{cm}} \qquad \omega \ll \Delta/h \qquad \alpha \gg \xi_0, \lambda_{\mathrm{L}} \tag{1}$$

are satisfied, then the electrodynamics of the Josephson junction is determined by the difference ϕ between the phases of the wavefunctions of the superconductors. In (1), V is the potential difference inside the junction, Δ is the energetic gap of the superconductors, h is Planck's constant, H_{cm} is the thermodynamic critical field of a massive superconductor, ξ_0 is the correlation length, and α and ω are the characteristic space period and frequency of the fields.

We will consider only dimensionless variables as in [2].

Under the conditions (1) the phase difference $\phi(x, y, t)$ inside the junction is determined by the sine–Gordon equation

$$\partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 - \partial^2\phi/\partial t^2 = \sin\phi. \tag{2}$$

The phase velocity connected with the wave operator on the left-hand side of the sine–Gordon equation (2) is the Swihart velocity c_0 [5], which is a major characteristic of the Josephson junction [1–4]. It is equal to unity in dimensionless variables.

The electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{H}$, the Josephson junction current density $\boldsymbol{j}_J$ and the potential difference V inside the junction are given by the equalities

$$\boldsymbol{E} = (\partial\phi/\partial t)\boldsymbol{n} \qquad \boldsymbol{H} = \boldsymbol{n} \times \nabla\phi \qquad \boldsymbol{j}_J = \boldsymbol{n}\sin\phi \qquad V = \partial\phi/\partial t \tag{3}$$

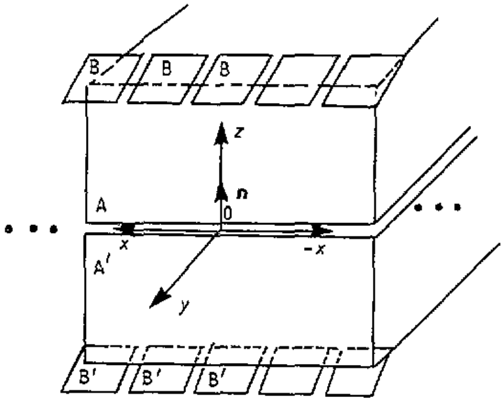

Figure 1. A scheme of the Josephson junction: A–A′, superconductors; B–B′, pairs of electrodes.

where $\boldsymbol{n}$ = unit vector along the z axis (see figure 1).

In [2] (see also [6]) a one-dimensional, semi-infinite Josephson junction is considered ($y \leqslant 0$). An external magnetic field H_0 is applied to the junction. The field is perpendicular to the y direction and it lies in the junction's plane (see figure 1).

When the value H_0 of the field satisfies the condition

$$H_0 < 2 \tag{4}$$

then the distribution of the phase difference ϕ is given by the expression

$$\phi(y) = 4\tan^{-1}[\exp(y + y_0)]. \tag{5}$$

The distribution of the magnitude H of the magnetic field corresponding to (5) inside the junction is given by the expression

$$H(y) = 2/\cosh(y + y_0). \tag{6}$$

The parameter y_0 is determined by the boundary condition

$$\cosh y_0 = 2/H_0. \tag{7}$$

The latter condition enables two values of y_0 to exist. The stable distribution of magnetic field corresponds to the value of y_0 at which the field H decays inside the junction. We will use that concept further in our considerations.

Of course, the one-dimensional case with respect to y could be regarded as the two-dimensional one in the xy plane, where the magnetic field H_0 is homogeneous with respect to the variable x. Our investigation generalizes this physical situation, transforming it into a new one, where a non-homogeneity with respect to x exists.

The aim of this paper is the following. On the basis of a specific exact solution of the sine–Gordon equation (2), we will investigate the resonance properties of a two-dimensional semi-infinite Josephson junction with respect to a weak external electric field in the form of a running wave with a phase velocity equal to the Swihart velocity. The external oscillating electric field is consistent with the exact solution. We will show that when such a field is applied along the junction's border then an electromagnetic structure appears inside. The structure moves with the Swihart velocity. One could say that the present investigation generalizes and moves along x (see figure 1) the homogeneous (with respect to x) stationary physical situation mentioned above and considered in [2] and [6].

The main features of the structure predicted are clarified. An experiment for verifying the existence of the structure is proposed. As a consequence, a resonance method for measuring the Swihart velocity is proposed. Some considerations concerning the stability of the structure investigated are presented.

2. Setting the problem and investigating the electromagnetic structure

We consider a Josephson junction situated in the half-plane $y \leqslant 0$ of the plane xOy (see figure 1). As in [1–4] we consider weak electromagnetic fields at small frequencies ω and wavenumbers k; for example, $\omega < 10^{11}$–10^{12} Hz, $k < 10^2$–10^3 cm^{-1}.

It has been established [7] that equation (2) possesses the solution

$$\phi(x, y, t) = 4\tan^{-1}[f(x-t)\mathrm{e}^y]. \tag{8}$$

Here $f(x-t)$ is an arbitrary function. The solution (8) describes a non-linear running wave moving along the direction x with a phase velocity equal to unity in dimensionless variables or equal to the Swihart velocity c_0 in physical variables. It may be seen from (3) and (8) that the function (8) describes some configuration of electric and magnetic fields as well as a superconducting Josephson current-density distribution inside the junction.

We will investigate the conditions for the appearance of the structure mentioned above and its properties.

From (3) and (8) we obtain for $\boldsymbol{E}$, $\boldsymbol{H}$ and $\boldsymbol{j}_{\mathrm{J}}$ inside the junction the following:

$$E_x = 0 \qquad E_y = 0 \qquad E_z = -4f_x/(\mathrm{e}^{-y} + f^2\mathrm{e}^y) \tag{9a}$$

$$H_x = -4f/(\mathrm{e}^{-y} + f^2\mathrm{e}^y) \qquad H_y = 4f_x/(\mathrm{e}^{-y} + f^2\mathrm{e}^y) \qquad H_z = 0 \tag{9b}$$

$$j_x = 0 \qquad j_y = 0 \qquad j_z = -4f\mathrm{e}^y(1 - f^2\mathrm{e}^{2y})/(\mathrm{e}^{-y} + f^2\mathrm{e}^y)^2 \tag{9c}$$

where

$$f_x \equiv \partial f(x-t)/\partial x.$$

The full superconducting current I is

$$I = \int_{-\infty}^{0}\int_{-\infty}^{\infty} j_z \,\mathrm{d}x\,\mathrm{d}y. \tag{10}$$

After the substitution $u \equiv x - t$ we obtain from (9c)

$$I = -4\int_{-\infty}^{\infty} \frac{f(u)}{1 + f^2(u)}\,\mathrm{d}u. \tag{11}$$

From (3) and (8) we have

$$V = -4f_u/(\mathrm{e}^{-y} + f^2\mathrm{e}^y) \tag{12}$$

where

$$f_u \equiv \mathrm{d}f(u)/\mathrm{d}u.$$

We shall consider the case when the fields (9a) and (9b) decay with $|y|$ analogously to the already mentioned stable and stationary one-dimensional case [2, 6]. So it may be said that we shall consider electromagnetic structures possessing Meissner-like properties.

The quantities $|H_x|$, $|H_y|$ and $|E_z|$ from (9) decrease with $|y|$ when

$$f^2(u) \leqslant 1. \tag{13}$$

At the boundary $y = 0$ for the fields (9a) and (9b) we obtain

$$E_{x_0} = 0 \qquad E_{y_0} = 0 \qquad E_{z_0} = -4f_u/(1+f^2) \tag{14a}$$

$$H_{x_0} = -4f/(1+f^2) \qquad H_{y_0} = 4f_u/(1+f^2) \qquad H_{z_0} = 0. \tag{14b}$$

Let us consider the following ordinary differential equation for $f(u)$:

$$-4f_u/(1+f^2) = h(u) \tag{15}$$

where $h(u)$ is an arbitrary function. The solution of equation (15) is

$$f(u) = \tan\left(-\tfrac{1}{4}\int_{-\infty}^{u} h(u)\,\mathrm{d}u + C\right) \tag{16}$$

where C is an arbitrary constant.

From (14) and (16) we have

$$E_{x_0} = 0 \qquad E_{y_0} = 0 \qquad E_{z_0} = h(u) \tag{17a}$$

$$H_{x_0} = -2\sin\left(-\tfrac{1}{2}\int_{-\infty}^{u} h(u)\,\mathrm{d}u + 2C\right) \qquad H_{y_0} = -h(u) \qquad H_{z_0} = 0. \tag{17b}$$

From (11) and (16) it follows that

$$I = 2\int_{-\infty}^{\infty} \sin\left(-\tfrac{1}{2}\int_{-\infty}^{u} h(u)\,\mathrm{d}u + 2C\right)\mathrm{d}u. \tag{18}$$

For the voltage $V(x, y, t)$ using (12) and (16) we obtain

$$V(x,y,t) = h(x-t)\,\mathrm{e}^{y}\left[\cos^2\left(-\tfrac{1}{4}\int_{-\infty}^{x-t} h(u)\,\mathrm{d}u + C\right) + \sin^2\left(-\tfrac{1}{4}\int_{-\infty}^{x-t} h(u)\,\mathrm{d}u + C\right)\mathrm{e}^{2y}\right]^{-1}. \tag{19}$$

It may be verified that the expressions (9) satisfy the Maxwell equations. If an electric field $\boldsymbol{E}$ in the form of (9a) exists inside the junction, it creates a magnetic field $\boldsymbol{H}$ and Josephson current density j_J in the form of (9b) and (9c). The electric field (9a) satisfies the boundary conditions (14a) and consequently, when an external electric field in the form (14a) is applied along the boundary $y = 0$, it is possible for an electromagnetic field in the form (9a) and (9b) to arise inside the junction.

From (13) and (16) it follows that

$$\left|-\tfrac{1}{4}\int_{-\infty}^{u} h(u)\,\mathrm{d}u + C\right| \leqslant \frac{\pi}{4}. \tag{20}$$

The value of the parameter C will be determined later.

Now we will consider some particular cases for the function $h(u)$.

(i) Let $h(u) \equiv 0$. From (17) we obtain

$$E_{x_0} = 0 \qquad E_{y_0} = 0 \qquad E_{z_0} = 0 \tag{21a}$$

$$H_{x_0} = -2\sin(2C) \qquad H_{y_0} = 0 \qquad H_{z_0} = 0. \tag{21b}$$

These boundary conditions correspond to the one-dimensional case mentioned above [2, 6].

(ii) There now follows an example of a function $h(u)$ that is different from zero in a finite interval. From (17) we have

$$\lim_{u\to\pm\infty} E_{z_0} = 0 \qquad \lim_{u\to\pm\infty} H_{y_0} = 0 \tag{22a}$$

$$\lim_{u\to-\infty} H_{x_0} = -2\sin(2C) \qquad \lim_{u\to\infty} H_{x_0} = -2\sin\left(-\tfrac{1}{2}\int_{-\infty}^{\infty} h(u)\,\mathrm{d}u + 2C\right). \tag{22b}$$

Let us impose on the function $h(u)$ the condition

$$\int_{-\infty}^{\infty} h(u)\,\mathrm{d}u = 0. \tag{23}$$

The boundary conditions (17) correspond to a junction to which an external constant magnetic field with components (21b) and an external electric field in the form (17a) along the boundary $y = 0$ are applied. For simplicity we regard the function

$$h(u) = \begin{cases} 0 & u < 0 \\ 1 & 0 \leqslant u \leqslant 1 \\ -\frac{1}{2} & 1 < u \leqslant 3 \\ 0 & 3 < u. \end{cases} \tag{24}$$

Let $C = 0$. From (18) and (24) we obtain

$$I = 12[\cos(\tfrac{1}{2}) - 1]. \tag{25}$$

In this consideration the length $3\lambda_J$ of the region where the function $h(u)$ is non-zero is an arbitrarily chosen model value and does not concern the main features of the phenomenon.

(iii) Now we shall consider an example of a periodic function $h(u)$. Let us accept for $f(u)$ a form slightly different from (16), i.e.

$$f(u) = \tan\left(-\tfrac{1}{4}\int_{0}^{u} h(u)\,\mathrm{d}u + C\right). \tag{26}$$

From (20) we obtain a condition similar to (23):

$$\int_{0}^{T} h(u)\,\mathrm{d}u = 0 \tag{27}$$

where T is the period of the function $h(u)$. In this case C cannot be determined from the boundary conditions. We assume that a non-stationary but equilibrium configuration of electromagnetic field exists inside the junction. The value of the parameter C will be estimated by a minimization of the thermodynamic potential. Considering constant temperature we minimize the free energy per period T. The calculations will be performed when

$$|h(u)| \ll 1. \tag{28}$$

For the free energy F of the junction we have [2]

$$F = \int_{-\infty}^{0}\int_{-\infty}^{\infty} [1 - \cos\phi + \tfrac{1}{2}(\nabla\phi)^2 + \tfrac{1}{2}(\partial\phi/\partial t)^2]\,\mathrm{d}x\,\mathrm{d}y. \tag{29}$$

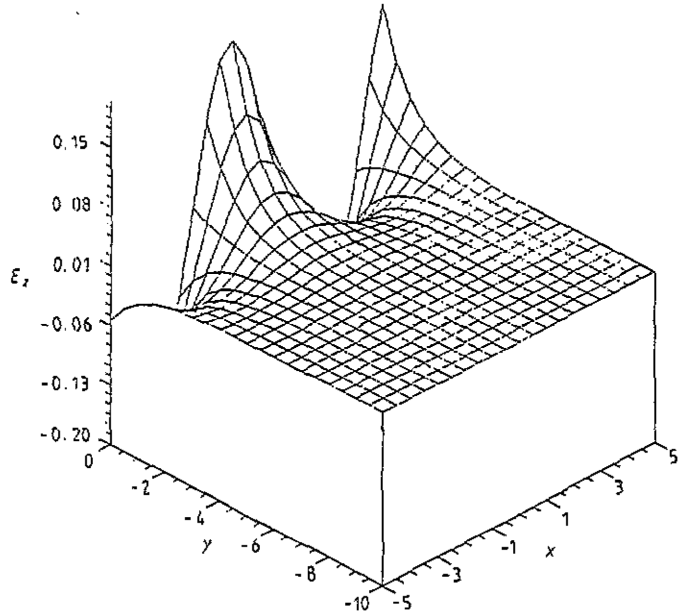

Figure 2. An illustration of the distribution of the component E_z of the electric field inside the junction when $h(x-t) = 0.2\sin(x-t)$ at the moment $t = 0$.

From (8) and (29) we obtain for the free energy per period F_T

$$F_T = 8\int_0^T \frac{f_u^2 + f^2}{1+f^2}\,\mathrm{d}u \tag{30}$$

or

$$F_T = \int_0^T \left[\tfrac{1}{2}h(u)\Big/ \cos^2\left(-\tfrac{1}{4}\int_0^u h(u)\,\mathrm{d}u + C\right) + 8\sin^2\left(-\tfrac{1}{4}\int_0^u h(u)\,\mathrm{d}u + C\right)\right]\mathrm{d}u. \tag{31}$$

Let $h(u)$ be the function

$$h(u) = \begin{cases} \varepsilon & u \in (2n,\ 2n+1] \\ -\varepsilon & u \in (2n+1,\ 2n+2]. \end{cases} \tag{32}$$

Here $n = 0, \pm 1, \ldots, 0 < \varepsilon \ll 1$. Using (32) and expanding the integrands in (31) into Taylor series keeping only the terms of the order of ε, we obtain

$$F_T = 8 - 8\cos(2C). \tag{33}$$

Obviously F_T is minimal when $C = 0$. The average value $\bar{j}_J$ of the superconducting current density per time period T is

$$\bar{j}_J = \frac{1}{T}\int_t^{t+T} j_z\,\mathrm{d}t. \tag{34}$$

From (9c), (32) and (34) we obtain

$$\bar{j}_J = 0. \tag{35}$$

Hence the average superconducting current across the junction is zero.

From these examples one can conclude that when the function $h(u)$ is in the form of $h(u) = -h(-u)$ then the full current across the junction is zero. The more the function differs from such a form, the more the full current differs from zero.

In figure 2 is presented an illustration of the distribution of the component E_z formed inside the junction when $h(x-t) = 0.2\sin(x-t)$ at $t = 0$.

3. A proposal for experimental verification

The existence of the predicted electromagnetic structure may be verified experimentally. For this purpose an external alternating electric field in the form (17a) must be applied along the border $y = 0$ of the junction. The total current is then continuously increased in *expectation* that a non-zero resonance potential difference appears on the superconductors not far from the border $y = 0$. This will prove that the discussed structure exists. It is well known that the photolithographic technique [8] allows electrodes with a thickness of 1000 Å to be evaporated. Such electrodes could be suitable for the measurement of the resonance. They must be distributed on the surfaces of the superconductors in the y direction. When the function $h(u)$ determining the external field satisfies the condition (20), then the potential difference $V(x, y, t)$ and the total current I are given by (19) and (18).

In practice, such an external electric field may be created in the following way. One has to situate the junction between pairs of electrodes set along the boundary (see figure 1). If we supply a voltage $V(x, t)$ in the form of

$$V(x, t) = H(x - t) \tag{36}$$

across the corresponding pairs of electrodes, then an electric field in the form (17a) arises along the boundary. Such a voltage may be obtained using a suitable delay line connected with the pairs. The voltage inside the junction will be determined by (19) if the width of the electrodes and the distance between them are much smaller than the characteristic length over which the function $H(u)$ changes.

The discussion above suggests a method for measuring the Swihart velocity (see [9]). There will be a resonance of the voltage inside the junction only when the phase velocity is equal to c_0. So scanning the phase velocity value of the external voltage when changing the total current provided, one must seek for an inside voltage resonance. Then measuring the delay τ after which a value V_0 of the external voltage appears on two pairs of electrodes situated at a distance λ from one another, one can find the Swihart velocity c_0 as

$$c_0 = \lambda/\tau. \tag{37}$$

As a model of a two-dimensional semi-infinite Josephson junction one could accept a construction consisting of two cylindrical coaxial superconductors (see figure 3). The external electric field runs around the dielectric barrier.

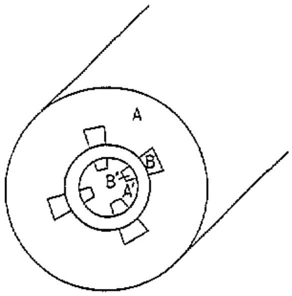

Figure 3. A scheme of a cylindrical Josephson junction: A–A′, superconductors; B–B′, pairs of electrodes.

4. Discussion

The investigation presented above points out that the Josephson junction possesses selective properties with respect to external electromagnetic fields in the form of a running wave with a phase velocity equal to the Swihart velocity.

The solution (8) originates from the solution (4). The solution (4) describes a stable distribution of the magnetic field. So there are grounds to expect that the structure predicted above is also a stable one.

Since the Swihart velocity is critical in our considerations, the electromagnetic structure is not of a type of fluxons.

The Swihart velocity was introduced for the first time [5] in the case of the absence of a Josephson current. Then the term $\sin\phi$ in the right-hand side of the sine–Gordon equation (2) is absent too. We call the value of the Swihart velocity thus calculated a linearly determined one. One could assume that when a Josephson current is available, then the value of the Swihart velocity differs from the linearly determined one.

The investigation presented above is to be understood in a more general sense. It points out the existence of a voltage resonance inside the junction when a fixed total current is provided and an electromagnetic field in the form of a running wave with a specific phase velocity is applied. So we assume that the real, non-linearly determined Swihart velocity is to be measured by the method proposed above.

5. Conclusion

Another solution of equation (15) is the function $v(u) = -1/f(u)$, where $f(u)$ is determined by (16). The structure described by this solution is physically equivalent to the one already considered but the directions of the components H_x and j_z are opposite.

The Josephson junction consisting of two superconducting layers with a thin barrier between them possesses selective properties with respect to a weak external electromagnetic field in the form of a running wave with a phase velocity equal to the Swihart velocity. An expression of this selectivity is the existence of the formation of an electromagnetic field inside the junction when a proper total current is provided. This formation is in the form of a running wave with the same velocity. A potential difference exists inside the junction.

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References

[1] Josephson B D 1965 *Adv. Phys.* **14** 419
[2] Kulik I O and Yanson I 1972 *The Josephson Effect in Superconducting Tunneling Structures* (Jerusalem: Israel Program for Scientific Translations)
[3] Barone A and Paterno G 1984 *Physics and Applications of Josephson Effect* (Moscow: Mir) (in Russian)
[4] Abrikosov A A 1987 *Principles of Metal Theory* (Moscow: Nauka) (in Russian)
[5] Swihart J C 1961 *J. Appl. Phys.* **32** 461
[6] Kulik I O 1966 *Zh. Eksp. Teor. Fiz.* **2** 1952 (in Russian)
[7] Ouroshev D, Martinov N and Grigorov A 1991 *J. Phys. A: Math. Gen.* **24** L527–8
[8] Oliner A (ed) 1978 *Acoustic Surface Waves* (Berlin: Springer)
[9] Young D R *et al* 1960 *Solid State Electron.* **1** 378